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The peculiarities of hot liquid droplets heating and evaporation

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ABSTRACT

The change of the thermal state of a sprayed liquid droplet is calculated using the method of combined analytical and numerical research, which requires the balance of energy flows, incoming to the droplet and outgoing from it. The method evaluates unsteadiness of heat and mass transfer processes and interaction, which occurs under the influence of the Stefan's hydrodynamic flow, radiant flow absorbed in semitransparent liquid and the Knudsen layer, which surrounds the droplets. The expedience of the thermal state's evaluation of dispersed liquid is verified using the parameter, expressed by the ratio of liquid's initial temperature on equilibrium evaporation temperature of droplets. As the above-mentioned parameter is less than 1, liquid is offered to be called "cold"; "warm", as the parameter equals 1 and "hot", as it exceeds 1. In each case the peculiarities of the thermal state change of sprayed liquid droplets are individual during an unsteady phase transformation mode. The characteristic curves, representing the change of transfer parameters, are determined for conductively heated droplets and when the Knudsen layer's influence is neglected. These curves join together variation of the parameters of the thermal state change and phase transformation for droplets of infinitesimal set of diameters, but with the same initial liquid temperature, as the droplets evaporate in gas with constant parameters. Deviations from the characteristic curves allow evaluating the influence of more complicated boundary conditions on the interaction of transfer processes.

1. Introduction

Liquid droplet is one of the essential links in many thermal technologies, based on heat and mass transfer in two-phase flows: liquid fuel combustion, gas purification, evaporation of liquid solutions, protection of various surfaces against high temperatures, etc. Many of the above-mentioned thermal technologies can be regarded as traditional. Though in present stage of technical development, as the requirements for reliability of technologies increase and new areas of application appear, a new important aspect has turned up: reliability of thermal technologies and wider practical application depend on the ability to control the intensity of liquid's phase transformations in complicated heat and mass transfer conditions. Highly developed interphase contact area in water sprays, efficient energetic interaction between gas and liquid phases, significant energy demand for droplet warming and evaporation allow ensuring reliable protection of surfaces [1]. Practical application of aerosol reactors based on small droplets is very wide [2]. Experiments prove the efficiency of water injection into air compressors of gas turbines [3], which is reached, as air compression process is set towards isothermal conditions and thus expenditure for external work is reduced. Recently the technologies, based on water injection, are being applied for humidifying of flue gas in order to clean it and utilize condensation energy.

It is normal to regulate the rate of phase transformations in two- phase flows by changing dispersity of liquid spray. It is necessary to ensure rapid and smooth evaporation process of water droplets in the thermal technologies under consideration. For that it is necessary to approach water spray towards a monodisperse flow, in which the rate of transfer processes is even and very high. Rapid technological development offers unexpected solutions, which actually approach thermal technologies to nano technologies. Experiments prove that water droplets pass to an explosive evaporation mode, when their mean mass temperature exceeds saturation temperature, corresponding to the medium parameters, by 18 degrees [4]. An explosive evaporation is evoked by an intensive energetic influence of a laser beam or by a sudden change of pressure in the vicinity of the droplet. The two-phase flow, constructed in such way, contains hot water droplets – their temperature is close to a saturation temperature, corresponding to a lower pressure medium.

Control of phase transformation's intensity according to dispersity of sprayed liquid is limited, if the goal is to ensure necessary amount of evaporated liquid and place of evaporation, as equipment operates in slightly variable mode. Multifactorial control opens wider control possibilities, as dispersity of droplets is combined with correctly chosen thermal state of sprayed liquid.

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Nomenclature

a	thermal diffusivity
B	Spalding transfer number
c_p	mass specific heat
C_l	droplet drag coefficient
D	mass diffusivity
Fo	Fourier number
G	mass flow rate
I	control time index
I_ω	spectral intensity of radiation
$I_{\omega,0}$	spectral intensity of radiation of a black body
k	ratio of specific heat capacities
k_ω	spectral index of absorption
Kn	Knudsen Number
l	mean free path of molecule
L	latent heat of evaporation
m	vapour mass flux density
M	molecular mass
n	number of the term in infinite sum
n_ω	spectral index of refraction
$\bar{n}_\omega$	spectral complex index of refraction
Nu	Nusselt number
p	pressure
Pe	Peclet number
Pr	Prandtl number
q	heat flux density
r	coordinate of a droplet
R	radius of a droplet
Re	Reynolds number
R_M	universal gas constant
Sc	Schmidt number
T	temperature
V	volume
w	velocity
Δw	slip velocity of a droplet in gas

Greek symbols

α_k	condensation coefficients
α_v	evaporations coefficients
η	dimensionless coordinate
λ	thermal conductivity
μ	dynamic viscosity
ρ	density
τ	time
φ	angle between the opposite direction of the normal to the surface element and the incident beam
χ_ω	spectral absorption coefficients
ψ	azimuthally angle
ω	wave number

Subscripts

c	convective
e	equilibrium evaporation
ef	effective
f	evaporation
g	gas
i	time index in a digital scheme
I	index of control time
j	index of droplet cross-section
J	index of droplet surface
k	iteration index in a digital scheme
Kn	Knudsen layer limit
m	modify; mass average
l	conductive
L	liquid
r	radiation
R	droplet surface
v	vapour
vg	vapour-gas mixture
ω	spectral
Σ	total
0	initial state
∞	far from a droplet

Superscripts

$+$	external side of a surface
$-$	internal side of a surface

In order to ensure such control, it is necessary to cognize the regularities of heat and mass transfer in droplets in wide range of boundary conditions of heat and mass transfer. It is necessary to take into account unsteadiness and interaction of heat and mass transfer in droplets and their vicinity, which occurs under the influence of many factors, such as: selective radiation, the Stefan's hydrodynamic flow, temperature and pressure drops in the Knudsen layer, slipping of droplets in gas, turbulence of a two-phase flow surrounding droplets, variable physical properties, etc. These aspects are important for modern research of the "droplet" problem.

The study [5] generalizes the results of the foremost researches of the "droplet" problem, as steady heat and mass transfer in droplet's vicinity is examined, as it evaporates in low-temperature medium. Introduction of the Spalding heat transfer parameter [6] made premises to evaluate the influence of phase transformations on convective heating of droplets using the principles of the similarity theory. Logarithmic and power functions, based on the Spalding heat transfer parameter, found wide application, as the intensity of convective heating of evaporating droplets is being calculated using the empirical equation, constructed for a solid sphere. It is possible to state, that the study [7] generalizes the achievements of modern research of the "droplet" problem, as radiant influence is neglected. The study [8] presents the method, which allows transforming nonlinear parabolic-type second degree energy equation, which describes the thermal state of a spherically symmetrical droplet, consisting of semitransparent liquid, evaporating and warming in a flow of monochromatic radiation, into the problem of unsteady heat transfer with known source function and presentation of this problem using an infinite sum of integral equations. Coherence of the external and internal "droplet" problems is shown; the necessity to formulate the boundary conditions in regards to unknown temperature of a droplet surface is presented; the conditions of the mode of a droplet's explosive evaporation are discussed. The peculiarities of modelling of radiation in fuel combustion systems are discussed in [9]. The models of spectral radiation [10–13] make presumptions for research of interaction of combined heat transfer processes. The influence of the shape of a symmetrical liquid's volume on radiant absorption is approved in the study [10]. Radiant absorption in water and fuel droplets in the presence of an asymmetric source is examined in the study [11]. The study [12] applied advanced conductivity methods for research of the influence of radiant absorption in droplets on their thermal state, as radiant emission inside a droplet is neglected. The influence of radiant absorption on droplet's temperature and evaporation process is validated.

The combined analytical numerical method [13–16] of the "droplet" problem's solution is more applicable, when it is necessary to evaluate the interaction of unsteady heat and mass transfer processes and the peculiarities of spectral radiation. The study [14] presents a thorough review and analysis of the proceedings regarding liquid fuel heating and evaporation, in which combined analytical, asymptotical and numerical methods are used. The method of the iterative research of unsteady heat and mass transfer in large semitransparent liquid droplets is developed in the studies [13,15,16]. The problem of evaluation of temperature and pressure drops in the Knudsen layer appears in modelling of small droplet's phase transformations [17–35]. The experiments prove that droplet evaporation rate depends on kinetics of heat and mass transfer on the interface between the droplet surface and vapour–gas mixture [17]. The boundary models of kinetic and diffusive evaporation are distinguished according to the value of the Knudsen number. In the first case (Kinetic evaporation model, $Kn \to \infty$) the changes of droplet size are determined by molecular processes near the droplet, as liquid vapour molecules contact with the surface of liquid. In the second case (Diffusive evaporation model, $Kn \to 0$) the processes of vapour diffusion in gas influence the changes of droplet size. In order to construct phase transformation models for medium values of the Knudsen number, it is necessary to compound the solutions of boundary cases in the limit of the Knudsen layer. The influence of the Knudsen layer on heat and mass transfer processes can serve as a factor for determination of a droplet size [18]. As the value of the Knudsen number is sufficiently small, vapour flow density from evaporating droplets depends on the Stefan's hydrodynamic flow and the rate of diffusion processes, which occur under the influence of liquid vapour's gradient [8]. As the value of the Knudsen number increases, the influence of kinetic processes near droplet surface increases as well. Presuming that vapour molecules do not collide in the Knudsen layer, for a spherical droplet with an isothermal surface velocities of molecules distribute according to the Grad function; the values of the ratio of gas and droplet surface temperature's difference on gas temperature and the ratio of sound velocity on phase transformation rate are significantly less than 1; temperature and pressure drops are defined by the system of algebraic equations in the studies [22,23]. This system of equations integrates heat and mass transfer parameters on the droplet surface, in gas, as well as in gas and vapour mixture; a modified Prandtl number and adiabatic exponent for gas and vapour mixture is applied.

The goal of this study is to release the analytical–numerical method of the "droplet" problem solution from limitation of a droplet size. The authors use the mathematical model of temperature and pressure drops, suggested in the study [23], for which the numerical scheme of an iterative solution was constructed. This numerical scheme matches with the "droplet" research method, developed in the study [15]. Evaporation of cold and hot water droplets is researched using a numerical experiment.

2. Problem formulation

Heat and mass transfer processes in droplets and their vicinity are interdependent; therefore internal and external "droplet" problems are solved coherently. Heat flow, supplied to the droplets, is distributed to energy flow that heats the droplets and energy flow, which is used for phase transformations. According to this distribution it is possible to select unsteady and equilibrium modes of liquid's phase transformations. If supplied energy is used only for evaporation of liquid, an equilibrium evaporation occurs at a specific temperature of a droplet surface $T_{R,e}$. In the case of equilibrium evaporation of liquid it is enough to solve the external "droplet" problem. As a droplet evaporates in a non-radiant medium, its external heat and mass transfer can be described by the system of nonlinear algebraic and first-degree differential equations:

$$\frac{\partial(\rho_L V_L)}{\partial\tau} = -4\pi R^2 m_v^+, \tag{1}$$

$$m_v^+ = \frac{D_{vg}}{T_{vg,R}}\frac{M_v}{R_M R} p \ln\frac{p - p_{v,\infty}}{p - p_{v,R}}, \tag{2}$$

$$Nu_f = (2 + 0.57 Re^{1/2} Pr^{1/3}) \cdot (1 + B)^{-0.7}, \tag{3}$$

$$\frac{dw_L}{d\tau} = \frac{3}{8}\frac{C_L}{R}\frac{\rho_g}{\rho_L}\frac{|w_L - w_g|}{(w_L - w_g)^{-1}}, \tag{4}$$

$$C_L = \frac{24 + 4.8 Re^{0.63}}{(1 + B)^{0.2} Re}, \tag{5}$$

$$B = \frac{c_{p,g}(T_g - T_R)}{L + q/m_v^+}. \tag{6}$$

The first equation connects the change of volume of a warming liquid with vapour flux density on a droplet surface. The second equation describes intensity of phase transformations on a droplet surface [8]. The direction of vapour flow is unambiguously defined by a logarithmic function, called the Stefan's algorithm. As pressure of liquid vapour near a droplet is higher than partial pressure of vapour in the gas mixture around the droplet, vapour flow is positive and the droplet decreases, as liquid evaporates. Otherwise vapour flow is negative and the droplet grows, as liquid vapour condenses on its surface. The third, fourth and fifth equations describe the intensity of droplet's convective heating [36], the change of droplet velocity and coefficient of drag [37], correspondingly. The influence of phase transformations on the latter parameters is evaluated by the functions of the Spalding heat transfer number, defined by the sixth equation. As a droplet evaporates in radiant medium, the change of its thermal state is influenced by energy of radiant flow, absorbed in semitransparent liquid. Its intensity can be calculated by solving the internal "droplet" problem, the solution of which is necessary in the case on unsteady evaporation.

Part of heat, supplied to a droplet, heats it during unsteady evaporation. Total heat flux density in a droplet defines heating intensity. In the case of interacting heat transfer by conduction and radiation in a droplet the total heat flux density is defined by the following system of nonlinear second degree differential and integral equations:

$$\vec{q}_\Sigma^{\,-}(r,\tau) = \vec{q}_l^{\,-}(r,\tau) + \vec{q}_r^{\,-}(r,\tau), \tag{7}$$

$$\rho_L c_{p,L}\frac{\partial T(r,\tau)}{\partial\tau} = \frac{1}{r^2}\frac{\partial}{\partial r}\left\{r^2\left[\lambda_L\frac{\partial T(r,\tau)}{\partial r} + q_r(r,\tau)\right]\right\}, \tag{8}$$

$$\left.\frac{\partial T(r,\tau)}{\partial r}\right|_{r=0} = 0, \tag{9}$$

$$q_r(s) = \int_0^\infty\int_0^{2\pi}\int_0^\pi I_\omega(s)\sin\varphi\cos\varphi\, d\varphi\, d\psi\, d\omega, \tag{10}$$

$$\frac{\partial I_\omega(s)}{\partial s} = \chi_\omega(s)\left[n_\omega^2(s) I_{\omega 0}(s) - I_\omega(s)\right], \tag{11}$$

$$\bar{n}_\omega = n_\omega - ik_\omega. \tag{12}$$

The system of Eqs. (7)–(12) is analytically transferred into the system of integral equations [13], as temperature of an evaporating droplet is set to be known. The function $T_R(\tau)$, which defines variation of a droplet surface temperature, is being concretized according to the condition of energy flows on the droplet surface, which requires equivalence of all energy flows that enter the droplet surface and leave it at every time instant:

$$\vec{q}_\Sigma^{\,+}(\tau) = \vec{q}_\Sigma^{\,-}(\tau) + \vec{q}_f^{\,+}(\tau). \tag{13}$$

Temperature of a droplet surface changes during its unsteady evaporation. In the case of equilibrium evaporation behaviour of the droplet surface temperature depends on the boundary conditions of heat and mass transfer [15].

The equation (13), which defines the balance of energy flows on an evaporating droplet surface unambiguously connects the solutions of the external "droplet" problem, described by the system of Eqs. (1)–(6) and the internal "droplet" problem, described by the system of Eqs. (7)–(12). The Eq. (13) enables to formulate the conditions of unambiguousity for the complex "droplet" problem in regards to yet unknown temperature of the droplet surface:

$$T(r,0) = T_0(r), \quad T(R,0) = T_{R,0}, \quad T(R,\tau) = T_R(\tau). \tag{14}$$

As the influence of the Knudsen layer on heat and mass transfer processes can be neglected, temperature of vapour and gas mixture in the Eq. (2) is set to be equal to the droplet surface temperature and partial pressure of vapour near the droplet surface is set to be equal to pressure of saturated vapour in regards to the droplet surface temperature:

$$T_{vg,R}(\tau) \equiv T_R(\tau), \quad p_{v,R}(\tau) \equiv p_{s,R}(\tau). \tag{15}$$

As the conditions (15) are valid, the solution of the "droplet" problem (1)–(14) is unambiguous in regards to the droplet surface temperature.

According to the method, presented in [22,23], temperature and pressure drops in the Knudsen layer around the droplet can be defined by the system of algebraic equations:

$$\left(\frac{\frac{R}{R_{Kn}}}{1-\frac{R^2}{2R_{Kn}^2}} + \frac{8k_{vg}}{k_{vg}+1}\frac{Kn}{Pr_m}\right)\frac{T_R - T_{Kn}}{T_\infty} = \frac{8k_{vg}}{k_{vg}+1}\frac{Kn}{Pr_m}\left(\frac{T_R - T_\infty}{T_\infty}\right)$$
$$+\frac{R}{R_{Kn}}\frac{k_{vg}-1}{k_{vg}+1}\frac{R_{\mu,v}}{R_{\mu,vg}}\frac{\sqrt{2\pi R_{\mu,vg}T_{Kn}}}{p}m_v^+, \tag{16}$$

$$\left(1-\frac{\alpha_k R^2}{2R_{Kn}^2}\right)\frac{\sqrt{(2\pi R_{\mu,v}T_\infty)}}{p_{v,\infty}}m_v^+ = \left(1-\frac{\alpha_k R^2}{2R_{Kn}^2}\right)$$
$$\frac{R_{Kn}}{R}\frac{4Kn}{Sc}\frac{P}{p_{g,\infty}}\sqrt{\frac{R_{\mu,vg}}{R_{\mu,v}}}\frac{p_{v,Kn}-p_{v,\infty}}{p_{v,\infty}} = \alpha_v\left(1+\frac{p_{s,R}-p_{v,\infty}}{p_{v,\infty}} - \frac{T_R - T_\infty}{2T_\infty}\right)$$
$$-\alpha_k\left[1+\frac{p_{v,Kn}-p_{v,\infty}}{p_{v,\infty}} - \frac{T_{Kn}-T_\infty}{2T_\infty}\right]. \tag{17}$$

A modified Prandtl number and adiabatic exponent are defined by the following expressions [25]:

$$Pr = \frac{k_{vg}R_{\mu,vg}\mu_{vg}}{(k_{vg}-1)\lambda_{vg}}. \tag{18}$$

$$\frac{k_{vg}+1}{k_{vg}-1}\frac{pR_{\mu,vg}}{\sqrt{2\pi R_{\mu,vg}T_{Kn}}} = \frac{k_v+1}{k_v-1}\frac{p_{v,Kn}R_{\mu,v}}{\sqrt{2\pi R_{\mu,v}T_{Kn}}} + \frac{k_g+1}{k_g-1}\frac{p_{g,Kn}R_{\mu,g}}{\sqrt{2\pi R_{\mu,g}T_{Kn}}}. \tag{19}$$

The value of the Knudsen number is calculated by dividing the mean free path of molecules by a droplet diameter. External radius of the Knudsen layer is proportional to a droplet radius and the mean free path of molecules:

$$R_{Kn} = R + \beta l. \tag{20}$$

As the internal radius of the Knudsen layer is calculated according to the empirical expression (20), it is recommended to use the value of the proportionality factor $\beta = 0.75$ [22] and to determine the length of molecule's free path in regards to the parameters of gas mixture far from a droplet [35]:

$$l = \frac{\mu_\infty}{2}\frac{\sqrt{(2\pi R_{\mu,\infty}T_\infty)}}{p}. \tag{21}$$

Temperature and pressure drops in the Knudsen layer are defined by transcendental equations. Their solutions are unambiguous, as the parameters of the droplet surface and gas mixture far from the droplet are determined.

One can notice that temperature and pressure drops in the Knudsen layer, as well as other parameters of heat and mass transfer in evaporating droplets can be calculated by numerical solution of the Eq. (13), using the method of iterations. Therefore the system of Eqs. (1)–(21) is consentaneous and free of restriction in regards to a droplet size.

3. Numerical solution

It is very complicated to solve the system of nonlinear equations (1)–(21) directly using the method of finite differences due to calculation of divergence of radiant flux density in the integral–differential equation (7) and the indeterminacies, related to optimal selection of the steps of temporal and spatial coordinates. Furthermore, particularity of the numerical scheme's algorithm can be influenced by the necessity to evaluate thoroughly the regularities of spectral optical characteristics of semitransparent liquid, which are peculiar for different liquids. The combined method of the analytical–numerical solution of the presented system of equation is developed further in this study. The goal of this method is the following: at the beginning the system of Eqs. (7)–(12), which defines the internal "droplet" problem, is analytically transformed into the system of integral equations and later on it is being solved by numerical methods, as convergence of the iterative scheme is easily controlled. The assumption of the iterative numerical research scheme allows solving energy (7) and radiant flux density (9) equations independently. Then radiant flux density in the Eq. (7) is set to be known function of time and droplet coordinate:

$$q_r(r,\tau) = f_{q_r}(r,\tau) \tag{22}$$

and the spectral radiant intensity in the Eq. (9) is unambiguously defined, as the unsteady temperature field of the droplet is set to be known function of time and droplet coordinate:

$$T_L(r,\tau) = f_{T_L}(r,\tau). \tag{23}$$

The functions (22) and (23) are being concretized in the iteration k according to the parameters of heat transfer in a droplet, calculated in the iteration $k-1$.

The replacement of the change of light ray intensity $dI_\omega(s)/ds$ by the change $dI_\omega(r)/dr$ allows analytically transforming the system of Eqs. (9)–(11) into an integral equation [13]. As radiant flux density is defined and the conditions (14) are valid, the differential–integral energy equation (7) is replaced by an infinite sum of integral equations [13].

A dimensionless coordinate is divided into $J-1$ number of parts:

$$\eta_j R \equiv \sum_{j=2}^{J}\Delta\eta_j R = R\sum_{j=2}^{J}\left(\eta_j - \eta_{j-1}\right). \tag{24}$$

A control range of time $\Delta\tau_I \equiv 0 \div \tau_I$ is predicted. Its division $\tau_{i=1} \equiv 0$ determines the initial thermal state of the droplet; the division $\tau_{i=I} \equiv \tau_I$ defines time instant, for which a new thermal state of the droplet is being calculated, which satisfies the condition:

$$\tau_I \equiv \sum_{i=2}^{I}\Delta\tau_i = \sum_{i=2}^{I}(\tau_i - \tau_{i-1}). \tag{25}$$

The numerical scheme for calculation of an evaporating droplet surface temperature at time instant τ_I is being constructed by concretising the Eq. (13). An energy flow is set to be positive, if its vector is directed towards the droplet surface. Spectral absorption coefficients of semitransparent liquids are finite. Therefore liquid practically absorbs no radiant flow on its surface. In such case modules of radiant flux densities $\vec{q}_r^{\,+}$ and $\vec{q}_r^{\,-}$ are close to each other; the directions of their vectors are opposite in regards to the droplet surface. Heat flux densities of thermal convection in gas $\vec{q}_c^{\,+}$ and in the droplet $\vec{q}_c^-$ are unambiguously defined by the difference between gas and droplet surface temperatures and by the gradient of temperature field in the droplet. Energy flux density $\vec{q}_f^+$ of phase transformations is unambiguously defined by the Stefan logarithm in the equation (2). The scheme of the numerical solution of Eq. (13) is written as follows:

$$\left(2+0.57Re_{I,k-1}^{1/2}Pr_{I,k-1}^{1/3}\right)\frac{\lambda_{vg,I,k-1}}{2R}\left(T_{\infty,I,k-1}-T_{R,I,k}\right) = \lambda_{ef,I,k}\frac{\partial T_l}{\partial r}\bigg|_{I,J}+\frac{D_{vg,I,k-1}}{T_{Kn,I,k}}\frac{L_{I,k}\mu_v}{R_\mu R}p\ln\frac{p-p_{v,\infty,I,k-1}}{p-p_{Kn,I,k}}. \quad (26)$$

Effective thermal conductivity:

$$\lambda_{ef,I,k}=\lambda_{L,I,k}\cdot\left[1.86+0.86\tan h\left(\frac{Pe_{L,I,k}}{30}\right)\right], \quad (27)$$

evaluates the influence of thermal convection on energy transfer in a droplet [7]. A temperature gradient in a droplet is calculated according to the model of unsteady temperature field, presented in the study [15]. According to this model it is necessary to take into account intermediate thermal states of the droplet, i.e. it is necessary to calculate temperatures $T_{R,i}$ for every time instant τ_i. It is expedient to set only one step of time change in the time division scheme (27), because it is necessary to neglect the change of the droplet size [15], when calculating temperature dynamics of the surface of a non-isothermal initial state droplet. Then $I = 2$ and $\sum_{i=2}^{I}(\tau_i-\tau_{i-1})\equiv\tau_I$. The gradient of unsteady temperature field for time instant τ_I is calculated by the following scheme:

$$\frac{\partial T_l}{\partial r}\bigg|_{I,J}=\frac{T_{R,I,k}}{T_{R,0}}\frac{dT_o}{dr}\bigg|_J+\frac{2\pi}{R^2}\sum_{n=1}^{\infty}n(-1)^n f_{n,I,j,k} \times\frac{2}{a_{I-1}+a_{I,k-1}}\left(\frac{R}{n\pi}\right)^2\left\{1-\exp\left[-\left(\frac{n\pi}{R}\right)^2\frac{a_{I-1}+a_{I,k-1}}{2}\tau_I\right]\right\}. \quad (28)$$

The function of time, coordinate and term of the sum $f_{n,I,j,k}$:

$$f_{n,I,j,k}=\frac{(-1)^n}{n\pi}\frac{T_{o,j}}{T_{R,0}}\frac{T_{R,I,k}-T_{R,0}}{\tau_I}-\sum_{j=2}^{J} \times\int_{\eta_{j-1}}^{\eta_j}\left(\frac{a_{I,k-1}}{R^2}\frac{T_{R,I,k}}{T_{R,0}}\frac{dT_0}{d\eta}\bigg|_j+\frac{q_{r,I,k-1}}{R\rho_{I,k-1}c_{p,I,k-1}}\right) \cdot\left[n\pi\eta\cdot\cos(n\pi\eta)-\sin(n\pi\eta)\right]\cdot d\eta. \quad (29)$$

evaluates the peculiarities of the initial non-isothermality of a droplet, the rate of the droplet surface temperature change and radiant absorption in the droplet. A local radiant flux density in a droplet is calculated by integration according to the wave number, using the rectangular method. Integration according to angle of incident light beam is performed by the Gauss method. The influence of light beam reflection on liquid surface and the Brewster's angle are evaluated.

The boundary of the Knudsen layer is defined by this radius:

$$R_{Kn,I,k}=R+\beta l_{I,k},\quad l_{I,k}=\frac{\mu_{\infty,I,k-1}}{2}\frac{\sqrt{2\pi R_{\mu,\infty,I,k-1}T_{g,I,k-1}}}{p}. \quad (30)$$

A modified adiabatic exponent of liquid vapour and gas mixture:

$$k_{vg,I,k}=\frac{F_{vg,I,k}+1}{F_{vg,I,k}-1}. \quad (31)$$

An auxiliary function F_{vg} is defined by the expression

$$F_{vg,I,k}=\frac{k_v+1}{k_v-1}\frac{f_{v,I,k}}{f_{vg,I,k}}+\frac{k_g+1}{k_g-1}\frac{f_{g,I,k}}{f_{vg,I,k}}, \quad (32)$$

in which the functions f_v, f_g and f_{vg}, evaluating the results of previous iteration, are calculated according to the united scheme

$$f_{z,I,k}=\frac{p_{z,I,k-1}R_{\mu,z,I,k-1}}{\sqrt{2\pi R_{\mu,z,I,k-1}T_{Kn,I,k-1}}}, \quad (33)$$

in which only the value the parameter z is different: $z\equiv v$ is for function f_v; $z\equiv g$ is for function f_g and $z\equiv vg$ is for function f_{vg}. Knudsen and modified Prandtl numbers:

$$Kn_{I,k}=\frac{l_{I,k}}{2R},\quad Pr_{m,I,k}=\frac{k_{vg,I,k}R_{\mu,vg,I,k-1}\mu_{vg,I,k-1}}{\left(k_{vg,I,k}-1\right)\lambda_{vg,I,k-1}}. \quad (34)$$

Then temperature on the boundary of the Knudsen layer is calculated:

$$T_{Kn,I,k}=T_{R,I,k}-\frac{T_{g,\infty}}{f_{R,I,k}}\left[\frac{8k_{vg,I,k}}{1+k_{vg,I,k}}\frac{Kn_{I,k}}{Pr_{m,I,k}}\frac{T_{R,I,k}-T_{g,\infty}}{T_{g,\infty}} +\frac{R}{R_{Kn,I,k}}\frac{k_{vg,I,k}-1}{k_{vg,I,k}+1}\frac{R_{\mu,v}m^+_{v,I,k-1}}{R_{\mu,vg,I,k-1}}\frac{\sqrt{2\pi R_{\mu,vg,I,k-1}T_{Kn,I,k-1}}}{p}\right] \quad (35)$$

here:

$$f_{R,I,k}=\frac{RR_{Kn,I,k}}{R^2_{Kn,I,k}-0.5R^2}+\frac{8k_{vg,I,k}}{k_{vg,I,k}+1}\frac{Kn_{I,k}}{Pr_{m,I,k}}. \quad (36)$$

Then pressure of vapour on the boundary of the Knudsen layer is calculated:

$$p_{v,Kn,I,k}=p_{v,\infty,I,k-1}\frac{T_{Kn,I,k}-T_{\infty,I,k-1}}{2T_{\infty,I,k-1}} +\frac{\alpha_v}{\alpha_k}\left(p_{s,R,I,k}-p_{v,\infty,I,k-1}\frac{T_{R,I,k}-T_{\infty,I,k-1}}{2T_{\infty,I,k-1}}\right) -\left(1-\frac{\alpha_k R^2}{2R^2_{Kn,I,k}}\right)\frac{\sqrt{2\pi R_{\mu,v}T_{\infty,I,k-1}}}{\alpha_k}m^+_{v,I,k-1}. \quad (37)$$

The presented iterative method of numerical research of phase transformations and thermal state change of dispersed semitransparent liquid is free of limitation regarding droplet size. The change of a droplet radius is controlled by selection of time τ_I. The change of droplet's mean mass temperature is performed additionally during intensive warming of liquid. When droplet surface temperature is calculated according to the scheme (26), other heat and mass transfer parameters for time instant τ_I are calculated simultaneously. The change of the droplet volume is calculated

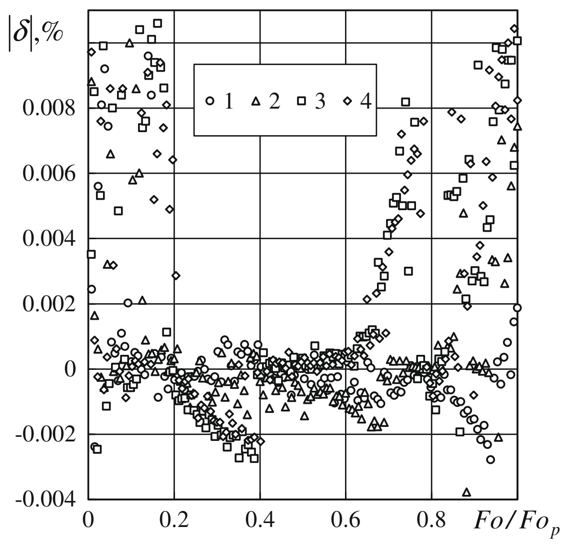

Fig. 1. Control of disbalance of energy flows on a droplet surface during a numerical experiment. T_g, K: (1) 473, (2) 673, (3) 873, (4) 1073; $R_0\cdot 10^6$, m: (1) 2, (2, 3) 5, (4) 10; $p = 10^5$ Pa; $p_{v,\infty}/p = 0.1$; $T_{R,0} = 370$ K; Fo_p – Fourier number of droplet lifetime.

by numerical solution of the Eq. (1), according to vapour flux density of phase transformations on the droplet surface. The boundary conditions (14) are specified: temperature field $T_0(\eta) \equiv T_L(\eta,\tau_I)$ of the droplet, calculated for time instant τ_I, is assigned to a new initial thermal state of the droplet. New control time τ_I is selected and the above mentioned calculations are repeated until a droplet evaporates out completely. If a single droplet evaporates, the changes in gas flow's parameters are not taken into account.

An infinite sum exists in the expression of the droplet's temperature field; indeterminacy is associated with this infinite sum. A numerical experiment was performed to determine an optimal number of the terms in the infinite sum: an unsteady temperature field in the droplet was calculated, evaluating various numbers of the terms in the infinite sum. It was determined that in order to calculate the most sensible parameter – local gradient of the temperature field, it might be necessary to evaluate the input of up to several hundred terms of the infinite sum. The sign of the term in the infinite sum depends on the term number: even and odd terms have opposite signs, if other parameters are similar. It is effective to use mean functions for adjacent terms of the infinite sum in the numerical scheme [38]. In such case number of terms of the infinite sum, close to 70, is the most optimal.

The analysis of droplet warming and evaporation process revealed a significant influence of liquid surface temperature on satisfying the condition of energy flow balance on the droplet surface, which evidences itself as a significantly indirect influence on the intensity of phase transformations. The instant value of a droplet surface temperature is being selected by iterative computations according to the scheme (26), using the method of the fastest descent in such a way, that it has to follow the condition of energy flow balance on the droplet surface at accuracy of one hundredth percent. Also it is necessary to evaluate the change of droplet surface temperature, which is smaller than one thousandth degree. As the influence of the Knudsen layer was neglected, the condition of energy balance on the droplet surface was satisfied at the accuracy of ±0.001% percent during major lifetime of the droplet. Under the influence of distinctively nonlinear effects in the Knudsen layer, a relative disbalance of energy flows on the droplet, equal to ±0.01%, is obtained during the initial and final stages of phase transformations. During the intermediary mode the condition (13) is satisfied at the accuracy of ±0.002% (Fig. 1).

4. Results and discussion

The influence of sprayed water‘s temperature on the thermal state of evaporating droplets in various cases of their heating is examined. A convective heating is modelled, as the initial velocity of droplet slip in gas is set above zero. In the case of combined heating an absolutely black source of spectral radiation having the same temperature as air, is present. Decreasing influence of external radiation on the interaction of combined heat transfer

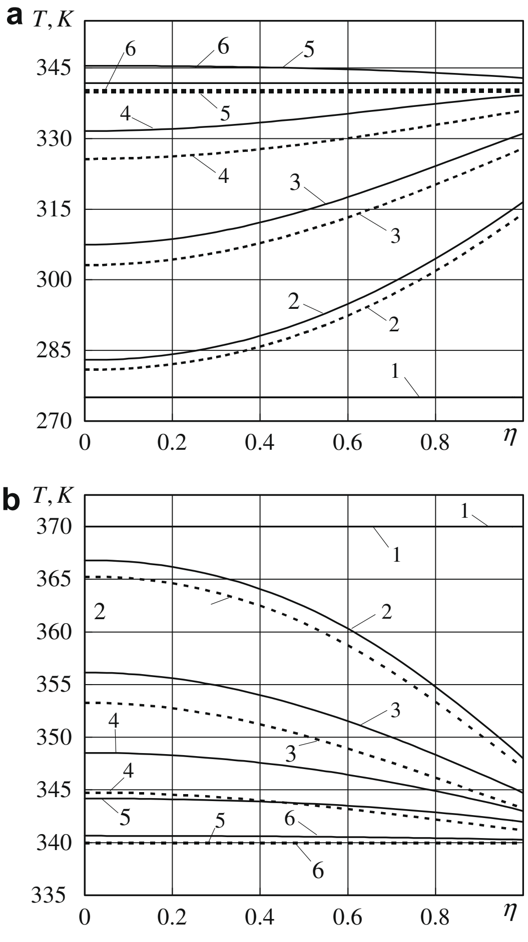

Fig. 2. Unsteady temperature field in cold (a) and hot (b) water evaporating droplet, as it is heated by conduction (dotted lines) and by conduction and radiation (continuous lines). Heating time τ, s: (1) 0, (2).006, (3).01, (4).02, (5).055, (6).16; $p = 10^5$ Pa; $p_{v,\infty}/p = 0.1$; $T_g = 1073$ K.

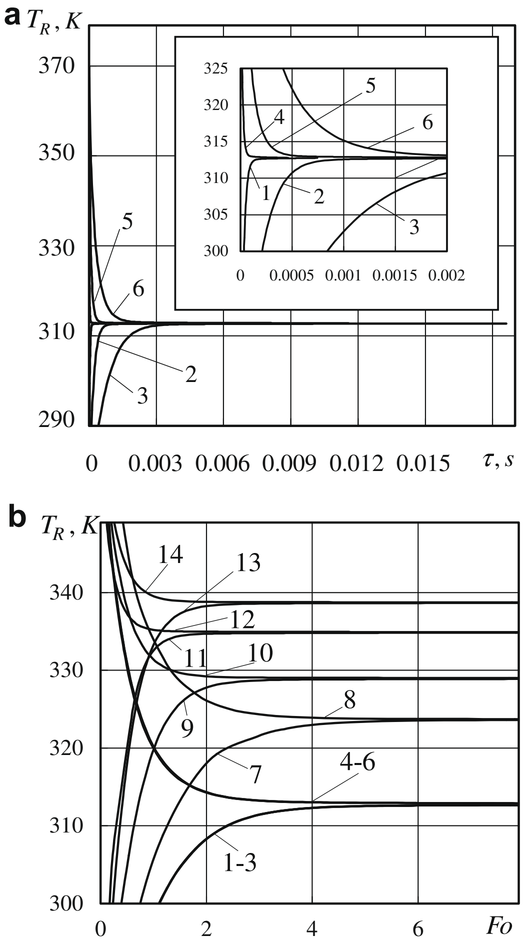

Fig. 3. Variation of surface temperature of conductively heated droplets in real (a) and Fourier number – based time scale (b). The influence of the Knudsen layer is neglected. $T_{R,0}$, K: (1–3, 7, 9, 11, 13) 275 (4–6, 8, 10, 12, 14) 370. $p = 10^5$ Pa; $p_{v,\infty}/p$: (1–6) 0, (7–12) .1, (13, 14) .2; T_g, K: (1–6, 9, 10, 13, 14) 473, (7, 8) 373, (11, 12) 673; $R_0 \cdot 10^6$, m: (1, 4) 2, (2, 5) 5, (3, 6) 10.

processes, as the droplet gets smaller, is evaluated by relating the lower boundary of radiant spectrum (in regards to the wave number) with a changing diameter of the droplet. Then, in the cases of liquid spraying into very small droplets or at the final evaporation stage of large droplets, radiation influence gradually weakens, though the Knudsen layer begins to make more and more significant influence.

Temperature of sprayed water determines the change of the thermal state of droplets, while the way of droplet heating determines the unsteady temperature field in the droplets (Fig. 2). Interphase contact $T_R(\tau)$ temperature plays a crucial role on the interaction of unsteady heat and mass transfer processes in liquid jets. This temperature determines phase transformations: when it is lower than a dew point, liquid vapour, which is present in gas, condenses on a droplet surface; when this temperature exceeds a dew point, evaporation of liquid occurs. The change of the thermal state of a non-isothermal droplet is the most precisely defined by dynamics of its mean mass temperature $T_m(\tau)$. Mean mass temperature ($T_{m,e,l}$) of a conductively heated droplet during its equilibrium evaporation is very important for evaluation of the unsteady temperature field's change of an evaporating droplet [39]. During the mode of equilibrium evaporation temperature $T_{m,e,l}(\tau) = T_{R,e,l}(\tau) \equiv T_{e,l}$. $T_{e,l}$ is influenced by air temperature and partial pressure of water vapour in air (Fig. 3). Though temperature $T_{e,l}$ does not depend on temperature of sprayed water. During the process of unsteady evaporation all droplets, for which Knudsen layer's influence is neglected, warm up or cool down to temperature $T_{e,l}$ (Fig. 3a). Then the change of the thermal state of the droplets with the same initial temperature is identical in time scale, expressed by Fourier number (Fig. 3b, curves 1–3 and 4–6). Calculated dynamics of the change of water droplets change $T_{m,l}(Fo)$ correlates with the experimental results of equilibrium evaporation's temperature of water droplets (Fig. 4).

The value of the ratio of the initial liquid temperature $T_{m,0}$ on temperature $T_{m,e}$ of liquid droplet that evaporates in equilibrium mode influences the change of the thermal state of sprayed liquid during unsteady phase transformations. As $T_{m,0}/T_{m,e} < 1$, droplets warm during unsteady phase transformations and it is possible to state that sprayed liquid is "cold". As $T_{m,0}/T_{m,e} > 1$, droplets cool during unsteady phase transformations and it is possible to state that sprayed liquid is "hot". As the value of temperature ratio $T_{R,0}/T_{R,e}$ is close to 1, the influence of unsteady phase transformation mode is negligible and the thermal state of the droplets

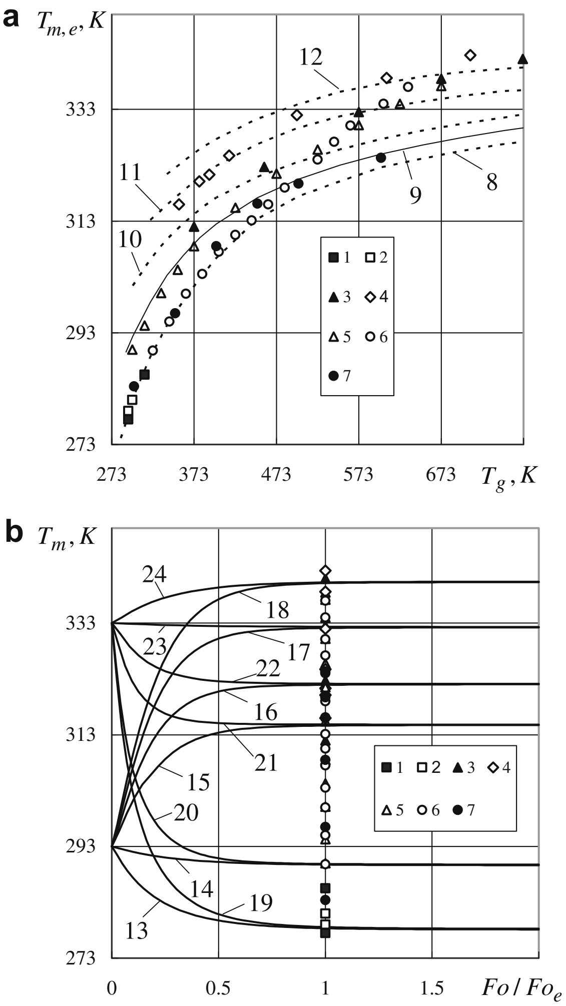

Fig. 4. Temperature of equilibrium evaporates water droplets. Points – the experiment [36,40]: (1) G. Langstrofh, (2) W. Ranz, W. Marschall, (3) A. Kobaijisi, (4) N. Nishiaki, (5) V. Fedoseev, D. Poliscuk, (6) M. Apasev, R.V. Malov, (7) C. Downing; dotted lines – numerical investigation [15], when $\Delta w \equiv 0$, $p_{v,\infty}/p$: (8) 0, (10) 0.05, (11) 0.1, (12) 0.15; (9) numerical investigation, when $p_{v,\infty}/p = 0.025$; continuous lines (b) – dynamics of mean mass temperature of conductively heated water droplet, when $\Delta w \equiv 0$, $p = 10^5$ Pa; $T_{m,0}$, K: (13–18) 293, (19–24) 333; $p_{v,\infty}/p$: (13, 14, 19, 20) 0, (15, 16, 21, 22) 0.05, (17, 23) 0.1, (18, 24) 0.15; T_g, K: (13, 19) 293, (14, 20) 323, (15, 21) 373, (16, 22) 473, (17, 23) 573, (18, 24) 773; Fo_e – Fourier number corresponding duration of droplet unsteady evaporation time.

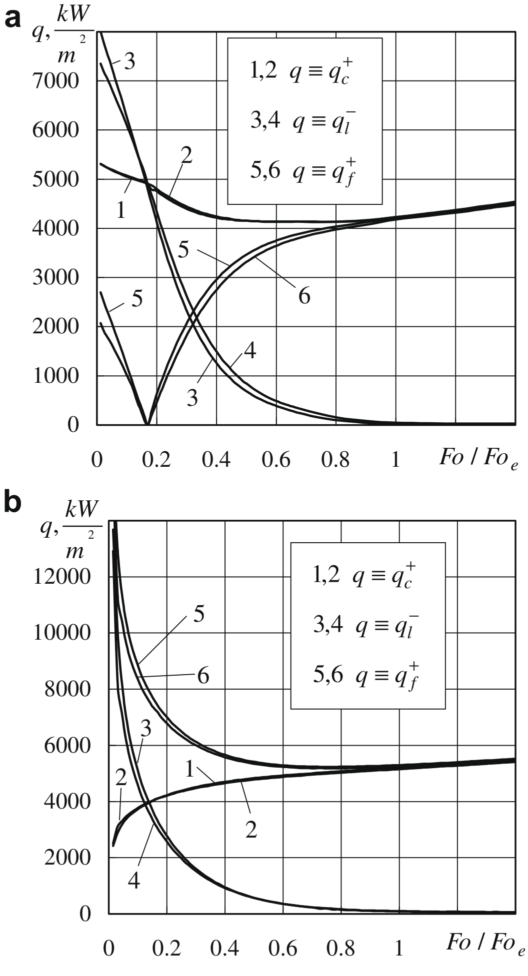

Fig. 5. Dynamics of energy flows on the surfaces of conductively heated "cold" (*a*) and "hot" (*b*) water droplets. (1, 3, 5) the influence of the Knudsen layer is neglected; $T_{R,0}$, K: (a) 275; (b) 370; $T_g = 873$ K; $p_{v,\infty}/p = .1$; Fo_e: (*a*) 1.38, (*b*) .85; $R_0 = 5 \cdot 10^{-6}$ m.

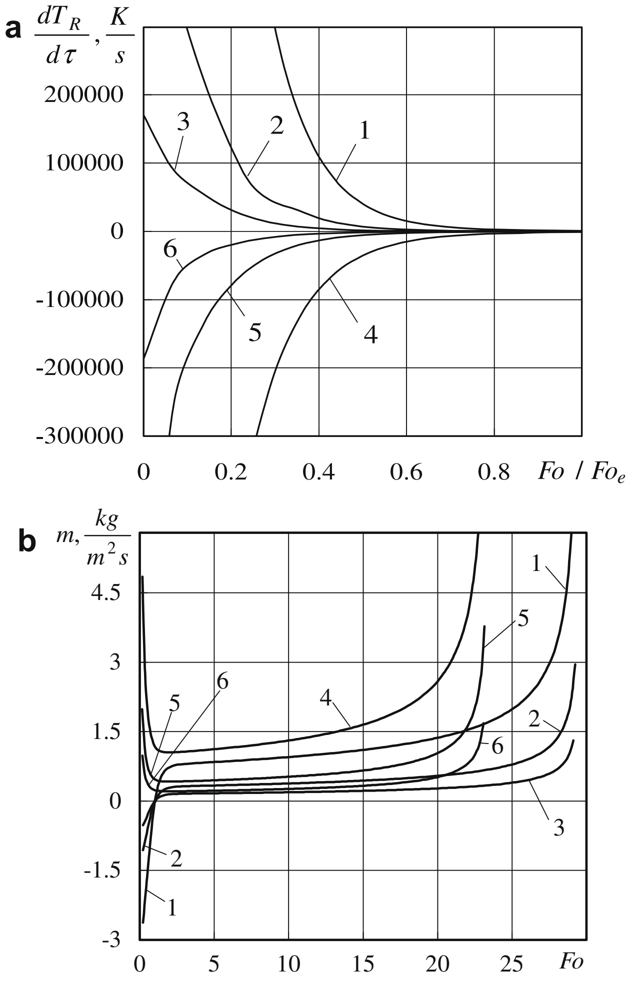

Fig. 6. Influence of water temperature on the warming rate (a) and the intensity of phase transformations of droplets (b). $T_{m,0}$, K: (1–3) 275, (4–6) 370 K; $R_0 \cdot 10^6$, m: (1, 4) 2, (2, 5) 5, (3, 6) 10; $p = 10^5$ Pa; $p_{v,\infty}/p = 0.2$; $T_g = 473$ K.

changes insignificantly during this mode. The flow of radiation, absorbed in water, accelerates warming of cold water droplets (Fig. 2a), but retards cooling of hot water droplets (Fig. 2b).

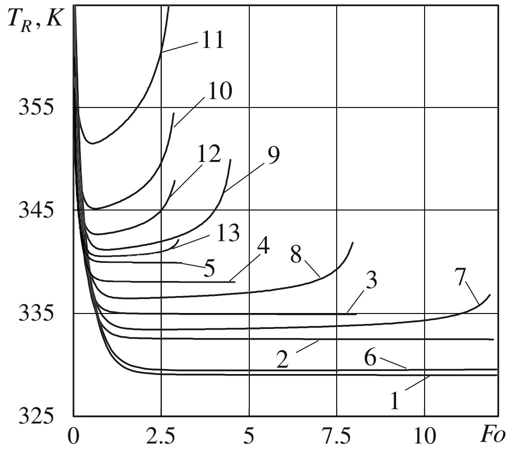

Fig. 7. The Knudsen layer's influence on the thermal state of "hot" water droplets. (1–5) are the characteristic curves of surface temperature variation; T_g K: (1, 6) 473, (2, 7) 573, (3, 8) 673, (4, 9) 873, (5, 10–13) 1073; $R_0 \cdot 10^6$, m: (6–10) 5; (11) 2; (12) 10; (13) 50; $T_{R,0} = 370$ K.

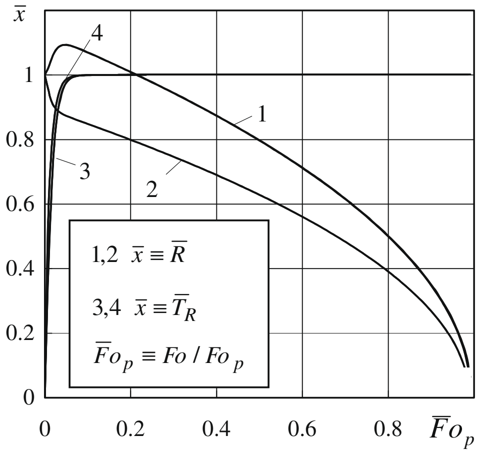

Fig. 8. The characteristic curves of water droplet evaporation dynamics (1, 2) and variation of droplet surface temperature (3, 4), as droplets are heated by conduction. $T_{R,0}$, K: (1, 3) 275, (2, 4) 370. $p_{v,\infty}/p$=0.2; T_g = 473K; $\overline{T}_R \equiv [(T_R - T_{R,0})/(T_{R,e} - T_{R,0})]$; $\overline{R} = R/R_0$; The influence of the Knudsen layer is neglected.

As already mentioned, at every time instant energy flows determine such temperature of a droplet surface, which ensures the balance of energy flows (Fig. 5), incoming to a droplet surface and

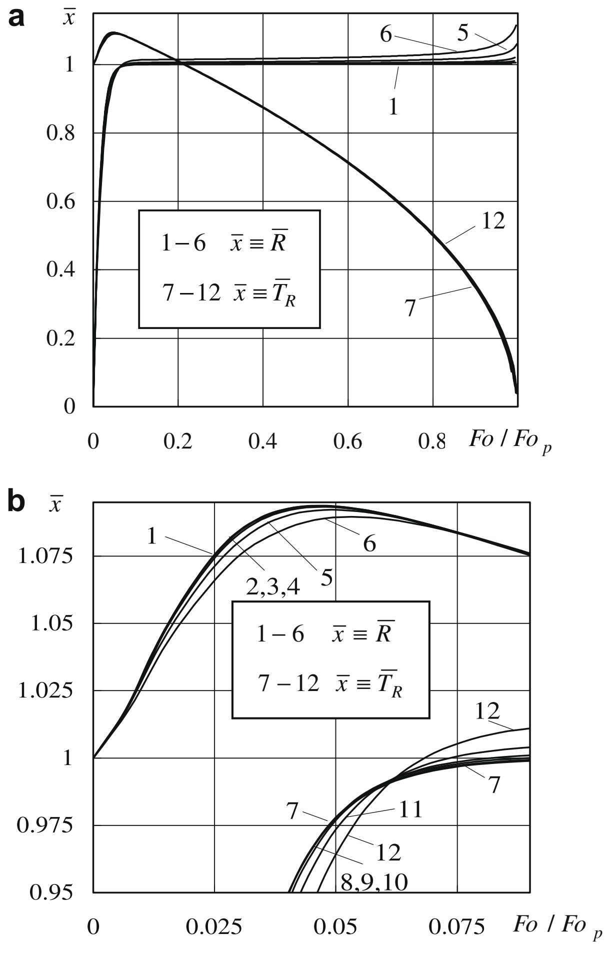

Fig. 9. The Knudsen layer's influence on the state of "cold" water droplets. (1, 7) are the characteristic curves, which correspondingly represent variation of droplet surface temperature and diameter. $T_{R,0} = 275$ K; $R_0 \cdot 10^6$, m: (2, 8) 20, (3, 9) 10, (4, 10) 5, (5, 11) 2, (6, 12) 1; $p_{v,\infty}/p = .2$; $T_g = 473$ K.

outgoing from it. Variations of surface temperature of “cold” and “hot” liquid droplets are peculiar, therefore dynamic of phase transformations in “cold” and “hot” droplets is essentially different (Fig. 6). “Cold” water droplets warm intensively during unsteady phase transformation. Heat, released in vapour condensation process, has sensible influence on warming rate of the droplet. As the droplet surface temperature approaches to a dew point, condensation intensity weakens, therefore warming rate gradually decreases (Fig. 6a). After conversion of phase transformation’s nature evaporation intensity gradually grows, as heating rate decreases till close to zero value during equilibrium evaporation stage. Condensation is impossible on the surface of “hot” droplets. They intensively cool during the mode of unsteady phase transformation (Fig. 6a); internal energy is used for evaporation of liquid. Therefore evaporation process, which was very intensive at the beginning, later on suddenly decreases, but at the final evaporation stage increases again due to rapid decreasing of the droplet size (Fig. 6b).

Temperature of sprayed liquid determines the peculiarities of energy flows on the droplet surface (Fig. 5). Heat flux density of phase transformations suffers significant qualitative changes. As liquid vapour is present in gas, a mode of vapour condensation on the droplet surface $0 \div Fo_{ko}$, specific only to “cold” liquid, exists, during which droplets warm intensively. The curve of conductive heat flux density $\vec{q}_l^-$, which represents warming intensity, during condensation mode of phase transformations lays over the curve of convective heat flux density $\vec{q}_c^+$, which represents the intensity of external heating of the droplet (Fig. 5a). Its vector is directed towards the droplet centre. At time instant of conversion of phase transformation’s nature the curve $\vec{q}_l^-$ crosses $\vec{q}_c^+$, but the directions of their vectors remain unchanged. As liquid is sprayed into dry gas, droplets immediately start evaporating, independently of initial temperature of liquid. But this temperature determines the change of evaporation intensity. The intensity of unsteady evaporation of “cold” liquid droplet rapidly increases. The curve, which represents this intensity $\vec{q}_f^+$ gradually approaches to the curve of convective heat flux density $\vec{q}_c^+$ from below (Fig. 5a). The intensity of unsteady evaporation of “hot” liquid droplet rapidly increases. The curve, which represents this intensity $\vec{q}_f^+$, gradually approaches to the curve of convective heat flux density $\vec{q}_c^+$ from above (Fig. 5b). The curve of conductive heat flux density $\vec{q}_l^-$, which represents cooling intensity of “hot” water droplet, lays below the curve $\vec{q}_f^+$, which represents heat flux density of phase transformations. Therefore, as distinct from the “cold” liquid case, only curves $\vec{q}_l^-$ and $\vec{q}_c^+$ cross with one another. Approaching to equilibrium evaporation conditions, the directions of the vector $\vec{q}_l^-$ can be the same, though they were opposite during the unsteady evaporation mode in the cases of “cold” and “hot” liquid. This is conditioned by warming of “hot” liquid droplet, which causes change of the direction of the vector $\vec{q}_l^-$ due to the influence of the Knudsen layer (Fig. 7). In general, independently of liquid temperature, transfer

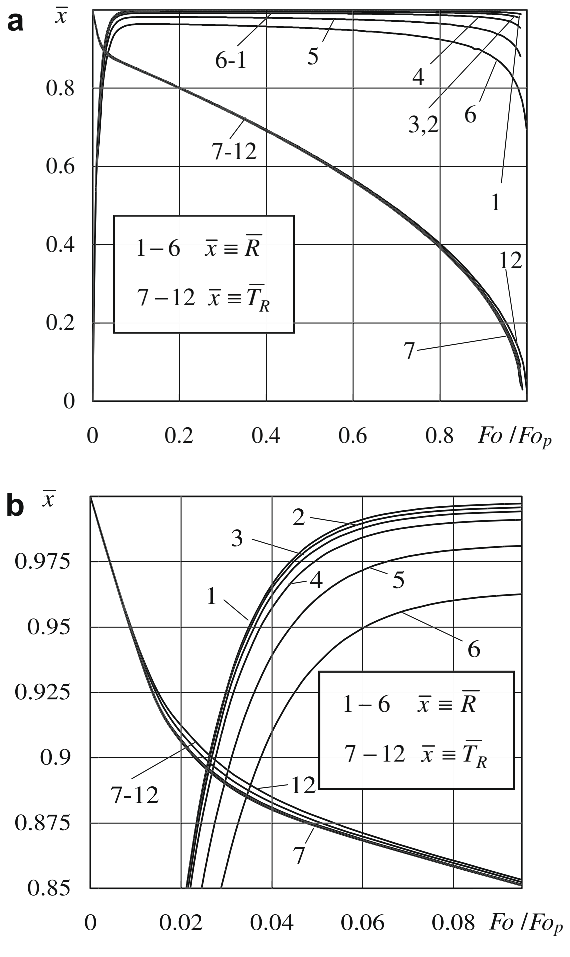

Fig. 10. The influence of gas temperature on transfer processes in the Knudsen layer in the case of “hot” water droplet evaporation. (1, 7) the characteristic curves, which correspondingly represent variation of droplet surface temperature and diameter. $T_{R,0}$ = 370 K; other boundary conditions correspond to Fig. 9.

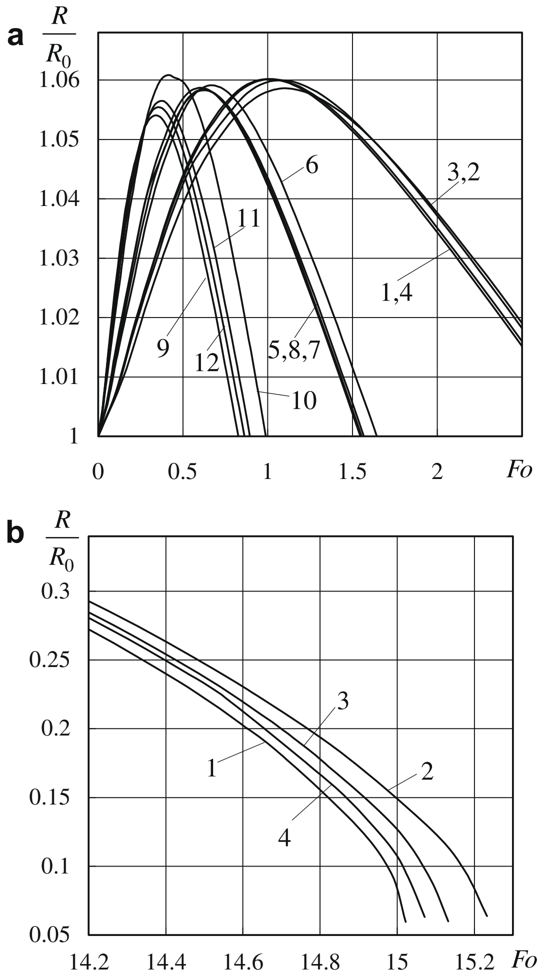

Fig. 11. The influence of gas temperature and droplet size on evaporation of sprayed “cold” water: (*a*) the initial stage of phase transformations; (*b*) the final stage of evaporation; (1, 5, 9) are the characteristic curves of droplet diameter variation; T_g, K: (1–4) 573, (5–8) 773, (9–12) 1073. $R_0 10^{-6}$, m: (2, 6, 10) 2, (3, 7, 11) 5, (4, 8, 12) 10; $p_{v,\infty}/p = 0.1$, $T_{R,0} = 275$ K.

effects in the Knudsen layer damp the intensity of transfer processes (Fig. 5, curves 5 and 6). Evaluation of the Knudsen layer makes no sensible effect on convective heating intensity (that is verified by the curves 1 and 2 of heat flux density $\vec{q}_c^+$), therefore the influence of similar level is transferred to conductive heat flux density in a droplet $\vec{q}_l^- = \vec{q}_c^+ + \vec{q}_f^+$. During the mode of unsteady evaporation module of the latter changes differently due to opposite directions of its vector: the module increases in a "cold" liquid droplet (Fig. 5a) and decreases in a "hot" liquid droplet (Fig. 5b).

The intensity of heat supplied to the droplets, which are carried by a non-radiating gas flow without slipping, directly depends on the droplet size. One can notice that vapour flux density on the surfaces of spherical droplets also directly depends on the droplet size. Thus a premise can be made for equilibrium evaporation of liquid to occur at constant temperature, as the droplets are heated by conduction and gas parameters are constant. As the influence of the Knudsen layer is neglected, it is possible to construct the characteristic curves, which show variation of the thermal state and dimensions of conductively heated droplets (Fig. 8). Dynamics of the thermal state of sprayed cold and hot water are similar, if it they are expressed by non-dimensional temperature $\overline{T}_R(\overline{Fo}_p)$. Peculiarity of the characteristic curves, which represent variation of non-dimensional size $\overline{R}(\overline{Fo}_p)$ of cold and hot water droplets, is influenced by intensive condensation of liquid vapour at the stage of cold droplet warming.

A universal curve should be able to join together the characteristic curves of a droplet surface temperature change $\overline{T}_{R,l}$ in duration of warming process, expressed by the ratio of Fourier numbers Fo/Fo_e. Reliability of this premise is partially verified by the digital experiments of unsteady phase transformations of "cold" water droplets with various initial temperatures [16]. Of course, it is

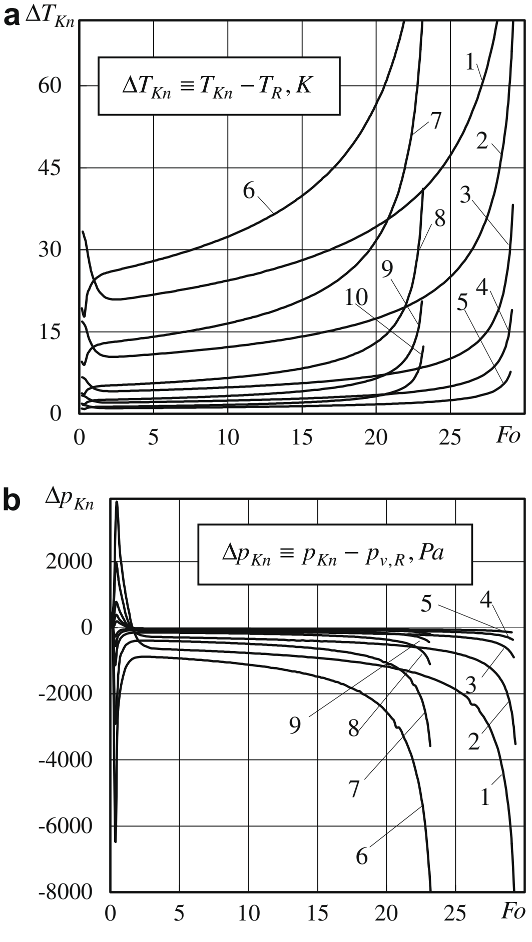

Fig. 12. Drops of temperature (*a*) and vapour pressure (*b*) in the Knudsen layer, surrounding "cold" (1–5) and "hot" (6–10) water droplets. $T_{R,0}$, K; (1–5) 275, (6–10) 370; $R_0 \cdot 10^6$, m: (1, 6) 1, (2, 7) 2, (3, 8) 5, (4, 9) 10, (5, 10) 20; $p_{v,\infty}/p$ = .2; T_g = 473 K.

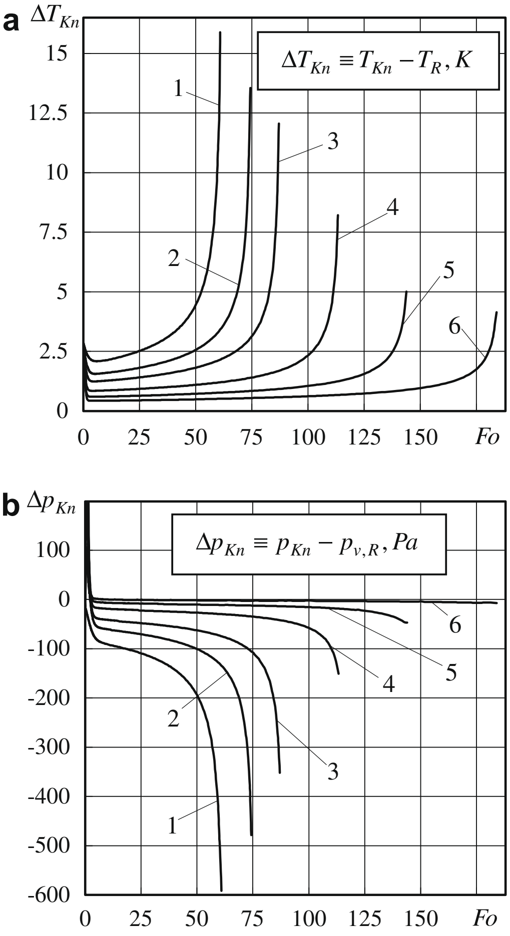

Fig. 13. The influence of gas humidity on temperature (*a*) and vapour pressure (*b*) drops in the Knudsen layer. $p_{v,\infty}/p$: (1) 0, (2).05, (3).1, (4).2, (5).3, (6).4; $T_{R,0}$ = 275 K; T_g = 373 K; $R_0 = 5 \cdot 10^{-6}$ m.

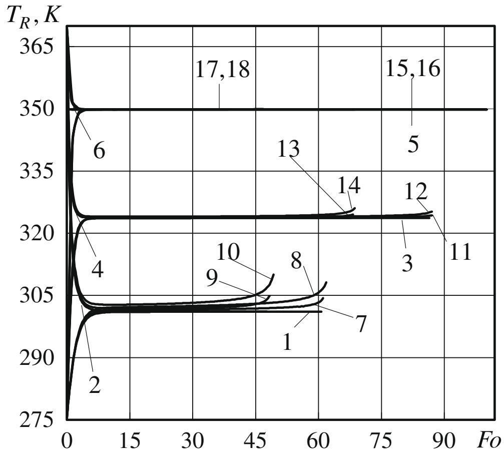

Fig. 14. The Knudsen layer's influence on the thermal state of water droplets, evaporating in low-temperature gas. (1–6) are the characteristic curves of surface temperature variation; $p_{v,\infty}/p$: (1, 2, 7–10).05, (3, 4, 11–14).1, (5, 6, 15–18).4; $T_{R,0}$, K: (1, 3, 5, 7, 8, 11, 12, 15, 16) 275, (2, 4, 6, 9, 10, 13, 14, 17,18) 370; $R_0 \cdot 10^6$, m: (7, 9, 11, 13, 15, 17) 5, (8, 10, 12, 14, 16, 18) 2; T_g = 373 K.

necessary to have the values of Fourier number, which determine duration of an evaporating droplet, and equilibrium evaporation temperature of liquid, as the droplets are heated by conduction. In order to obtain the universal curve of a droplet surface temperature change, which would be free of limitations of water initial temperature and gas parameters, it is necessary to determine additional corrective functions, which could evaluate the influence of the above-mentioned parameters. This problem requires a thorough evaluation of heat and mass transfer process interaction in wide range of boundary conditions. One can mention that it is possible to make conclusions, important from practical point of view according to deviation of the process parameters from the characteristic curves. Deviation of droplet warming and evaporation dynamics from the characteristic curves under the influence of the transfer processes in the Knudsen layer depends on initial temperature and dispersity of sprayed liquid (Figs. 9 and 10). As liquid is sprayed into gas with temperature 473 K, in which $p_{v,\infty}/p = 0.2$, the Knudsen layer's influence can be neglected for "cold" water droplets, larger than 10 μ (Fig. 9) and for "hot" water droplets, larger than 20 μ (Fig. 10). It is easy to detect the significant influence of gas temperature on transfer processes in the Knudsen layer. As water is sprayed into gas at temperature 1073 K, the influence of the Knudsen layer is sensible even for large droplets, diameter of which is around 100 μ (Fig. 7). The Knudsen layer's influence on evaporation dynamics is more vivid at the initial and the final stages of phase transformations of small droplets (Fig. 11). Their volume increases more during vapour condensation and droplet warming (Fig. 11a) under the influence of the Knudsen layer, besides, droplet lifetime increases (Fig. 11b).

The values of temperature and vapour pressure drops determine the influence of the Knudsen layer. Dynamics of temperature (Fig. 12a) and vapour pressure (Fig. 12b) is distinctive for "cold" and "hot" liquid droplets. The changes of temperature and vapour pressure in the Knudsen layer depend on a droplet size. The peculiarities of phase transformations at the initial stage of intensive change of the droplet thermal state determine qualitative changes of vapour pressure drops in the cases of "cold" and "hot" liquid. The effect of gas moistening, which actually softens the Knudsen layer's influence, was observed (Fig. 13). In dry gas with temperature 373 K the Knudsen layer's influence on the change of liquid thermal state can be neglected for droplets, diameter of which is larger than 10 μ (Fig. 14). As the value of partial pressure of vapour in gas exceeds $p_{v,\infty}/p = 0.1$, the Knudsen layer has no influence on the change of liquid thermal state for droplets, diameter of which is larger than 1 μ.

The changes of the droplet heating mechanism have the essential influence on droplet warming dynamics (Fig. 15) and variation of energy flows on its surface (Fig. 16). As a large 200 μ "cold" water droplet gets into radiant gas at temperature 1073 K, the duration of their phase transformations shortens up to 15%; if velocity of droplet slip in gas is present, which corresponds to $Re_0 = 50$ – even about 45%, if compared to the conductive heating case.

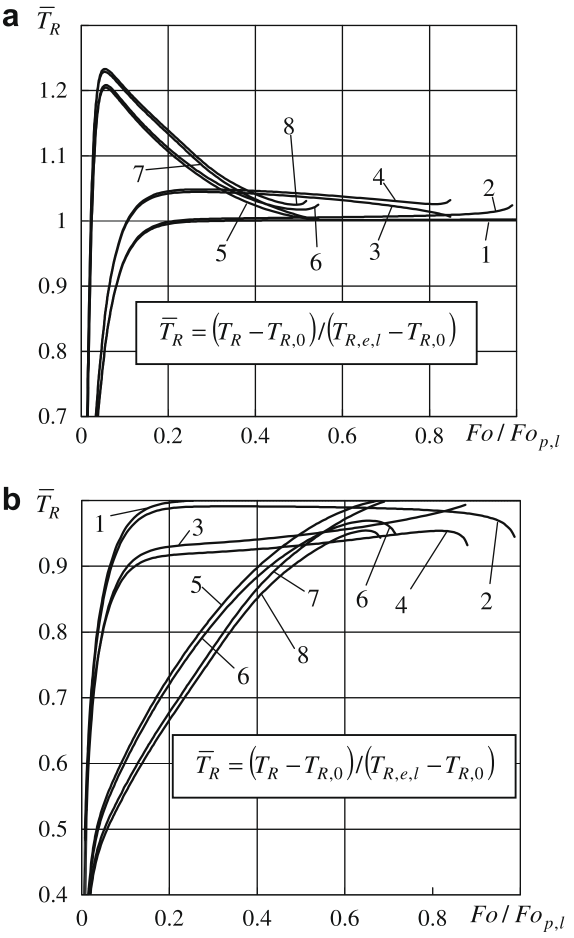

Fig. 15. The influence of radiant absorption in droplets and velocity of their slip in gas on the thermal state of "cold" (*a*) and "hot" (*b*) water droplets. (1,3,5,7) The influence of the Knudsen layer is neglected; (1,2,5,6) $q_r = 0$; $T_{R,0}$, K: (*a*) 275; (*b*) 370; Re_0: (1–4) 0; (6–8) 50; $Fo_{ko,l,Kn=0}$: (*a*) .112, (*b*) 0; $Fo_{e,l,Kn=0}$: (*a*) 1.13, (*b*) .78; $Fo_{p,l,Kn=0}$: (*a*) 3.84, (*b*) 3.11; $T_g = 1073$ K; $p_{v,\infty}/p = .1$; $R_0 = 100 \cdot 10^{-6}$ m.

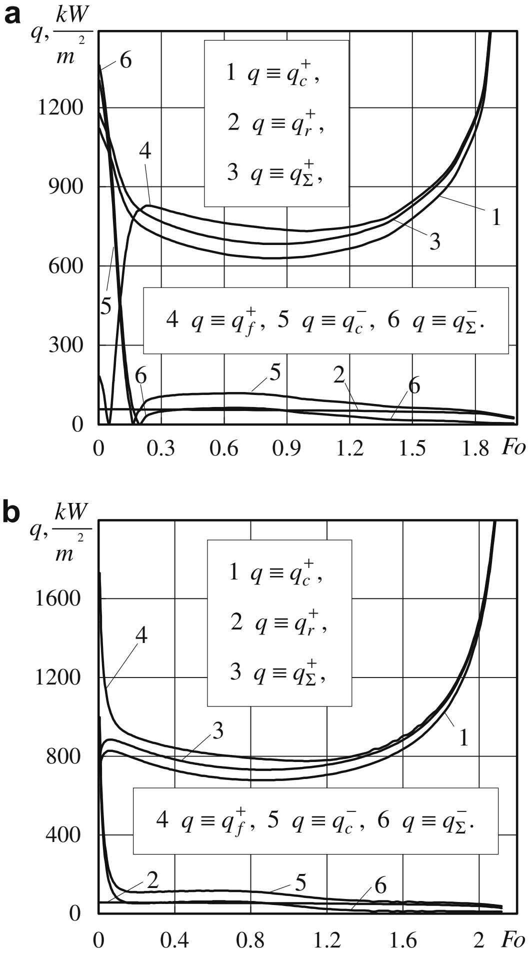

Fig. 16. Variation of energy flows on the surface of "cold" (*a*) and "hot" (*b*) water droplets in the case of radiant– convective heating. $T_{R,0}$, K: (*a*) 275; (*b*) 370; $Re_0 = 50$; $T_g = 1073$K; $p_{v,\infty}/p = .1$; $R_0 = 100 \cdot 10^{-6}$ m.

5. Conclusion

Initial temperature of sprayed liquid determines qualitative changes of temperature and vapour pressure in the Knudsen layer. The influence of temperature and pressure drops in the Knudsen layer on the interaction of heat and mass transfer depends on dispersity of sprayed liquid – it is more significant in the case of small droplets. Independently of initial dispersity the influence of the Knudsen layer must be evaluated at the final stage of phase transformations, as droplets decrease during evaporation process.

The characteristic curves, which represent variation of phase transformations, as droplets are heated by conduction, make good presumptions of comparative evaluation of combined heat and mass transfer interaction. One may designate two clear boundary cases of small and large droplets, carried by gas without slipping. In the first case deviation from the characteristic curves can be taken into account only by evaluation of the Knudsen layer's influence; in the second case – by evaluation of the peculiarities of radiant absorption in a droplet. Velocity of droplet slip in gas determines the influence of convective heating of droplets. In order to distinguish the influences of separate factors on the thermal state change of sprayed liquid and intensity of phase transformations in the case of combined heating of medium size droplets, more thorough research is necessary.

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